MATHS BASE MAKER

Class 9 CBSE Mathematics | Ganita Manjari

Chapter 4: Exploring Algebraic Identities

STUDENT PRACTICE WORKSHEET (concept + questions)

PART 1: MASTER LIST OF ALGEBRAIC IDENTITIES

1. Linear Product	$(x + a)(x + b) = x^2 + (a + b)x + ab$
2. Square of Binomial (Sum)	$(x + y)^2 = x^2 + 2xy + y^2$
3. Square of Binomial (Diff)	$(x - y)^2 = x^2 - 2xy + y^2$
4. Difference of Two Squares	$(x + y)(x - y) = x^2 - y^2$
5. Square of Trinomial	$(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$
6. Cube of Binomial (Sum)	$(x + y)^3 = x^3 + 3x^2y + 3xy^2 + y^3 = x^3 + y^3 + 3xy(x + y)$
7. Cube of Binomial (Diff)	$(x - y)^3 = x^3 - 3x^2y + 3xy^2 - y^3 = x^3 - y^3 - 3xy(x - y)$
8. Sum & Difference of Cubes	$x^3 + y^3 = (x + y)(x^2 - xy + y^2)$ $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$
9. Three-Variable Cubic Identity	$x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$ (Special condition: If $x + y + z = 0$, then $x^3 + y^3 + z^3 = 3xyz$)
10. Geometric Model of $(a + b)^2$	Area of square of side $(a + b) = a^2 + 2ab + b^2$ (Square a^2, two rectangles ab, and square b^2)

PART 2: PRACTICE QUESTIONS (WITH FINAL ANSWERS)

Watch the Complete Chapter 4 Video Solutions on YouTube PART 1

https://youtu.be/k8tPcPowyb8

contd..part 2 , 3 on youtube

Question 1: Expansion Using Suitable Identities

Expand each of the following expressions using appropriate algebraic identities:

(i) $(10x + 7b)^2$

(ii) $(\frac{1}{2}x - \frac{1}{2}y)^2$

(iii) $(204)^2$

(iv) $(97)^2$

(v) $(4a - 3b + 2c)^2$

Final Answer: **(i) $100x^2 + 140bx + 49b^2$**

(ii) $\frac{1}{4}x^2 - \frac{1}{2}xy + \frac{1}{4}y^2$

(iii) 41,616 $[(200 + 4)^2 = 40000 + 1600 + 16]$

(iv) 9,409 $[(100 - 3)^2 = 10000 - 600 + 9]$

(v) $16a^2 + 9b^2 + 4c^2 - 24ab - 12bc + 16ca$

Question 2: Factorisation Using Identities

Factorise each of the following expressions completely:

(i) $25y^2 - 20y + 4$

(ii) $\frac{1}{4}x^2 + 2xz + 4z^2$

(iii) $4a^2 + 9b^2 + c^2 - 12ab + 6bc - 4ac$

(iv) $10p^4 - 2560q^4$

Final Answer: **(i) $(5y - 2)^2$**

(ii) $(\frac{1}{2}x + 2z)^2$

(iii) $(-2a + 3b + c)^2$ $[\text{or } (2a - 3b - c)^2]$

(iv) $10(p^2 + 16q^2)(p + 4q)(p - 4q)$

Question 3: Quadratic Trinomial Factorisation

Factorise the following quadratic polynomials by splitting the middle term:

(i) $3x^2 - x - 4$

(ii) $x^2 - 9x + 20$

Final Answer: **(i) $(3x - 4)(x + 1)$**

(ii) $(x - 5)(x - 4)$

Question 4: Evaluation Without Direct Multiplication

Evaluate each of the following products using algebraic identities:

(i) $(106)^2 - (94)^2$

(ii) $(225)^2 - (175)^2$

(iii) 55×45

Final Answer: **(i) 2,400 $[(106 + 94)(106 - 94) = 200 \times 12]$**

(ii) 20,000 $[(225 + 175)(225 - 175) = 400 \times 50]$

(iii) 2,475 $[(50 + 5)(50 - 5) = 50^2 - 5^2 = 2500 - 25]$

Question 5: Completing Identities (Fill in the Blanks)

Fill in the blanks to complete the algebraic identities:

(i) (_____)(a + 2) = $3a^2 + 8a + 4$

(ii) $6x^2 - 5x - 6 = (3x + ____)(2x - 3)$

Final Answer: **(i) 3a + 2**

(ii) 2

Question 6: Binomial Square Expansions with Decimals

Expand each of the following binomials:

(i) $(4x - 3y)^2$

(ii) $(0.6a + 0.4b)^2$

Final Answer: **(i) $16x^2 - 24xy + 9y^2$**

(ii) $0.36a^2 + 0.48ab + 0.16b^2$

Question 7: Cubic Factorisations & Special Identities

Factorise each of the following expressions completely:

(i) $8x^3 + (1/27) + 4x^2 + (2/3)x$

(ii) $64a^3 - 24a^2 + 3a - (1/8)$

(iii) $27m^3 - (1/512)\, n^3$

(iv) $343p^3 + 216q^3$

(v) $p^3 + 27q^3 + r^3 - 9pqr$

Final Answer: **(i) $(2x + 1/3)^3$**

(ii) $(4a - 1/2)^3$

(iii) $(3m - 1/8\, n)(9m^2 + 3/8\, mn + 1/64\, n^2)$

(iv) $(7p + 6q)(49p^2 - 42pq + 36q^2)$

(v) $(p + 3q + r)(p^2 + 9q^2 + r^2 - 3pq - 3qr - pr)$

Question 8: Side of a Cube from Volume Expression

What is the side length of the cube whose volume is represented by the polynomial:

$V = 8x^3 - 60x^2y + 150xy^2 - 125y^3$?

Final Answer: **Side = 2x – 5y [Since $V = (2x - 5y)^3$]**

Question 9: Simplifying Rational Expressions

Simplify each of the following rational expressions using suitable identities:

(i) $(27a^3 - 64b^3) / (9a^2 - 16b^2)$

(ii) $(x^3 + 125y^3) / (x^2 - 5xy + 25y^2)$

(iii) $(4x^2 - 12x + 9) / (4x^2 - 9)$

(iv) $(x^4 - 256) / (x^2 - 8x + 16)$

Final Answer: **(i) $(9a^2 + 12ab + 16b^2) / (3a + 4b)$**
(ii) $x + 5y$
(iii) $(2x - 3) / (2x + 3)$
(iv) $(x^2 + 16)(x + 4) / (x - 4)$

Question 10: Symmetric Sum of Cubes Problem

The sum of three numbers is 12 and their product is 38. The sum of their squares is 60. Find the sum of the cubes of these three numbers.

Final Answer: **Sum of cubes = 432 $[x^3 + y^3 + z^3 = 432]$**

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part 2, 3 on youtube
