MATHS BASEMAKER
Class 9 CBSE — Ganita Manjari (Part II) — Chapter 9

Propositions and their Converses (conceps & imp questions)

(Strengthen your base with CBSE teacher Pradeep Sahajwani, having 19 years of experience)

What is a Proposition?

• Definition: A proposition is a declarative mathematical statement that is either strictly TRUE or strictly FALSE, but not both.

• Standard Form: "If X, then Y" (or "X implies Y").

- X (Hypothesis / Condition): The starting assumption.

- Y (Conclusion): The result that follows from X.

• Example 1 (True Proposition): "If a quadrilateral is a square, then all four of its angles are right angles (90°)."

• Example 2 (False Proposition): "If a number is divisible by 6, then it must end in zero."

What is a Converse & How to Frame It?

• Definition: The converse of a proposition is formed by interchanging (swapping) the Hypothesis (X) and Conclusion (Y).

• Original Proposition: "If X, then Y"

• Converse Statement: "If Y, then X"

• Framing Practice:

- Original: "If $x = y$, then $x^2 = y^2$."

- Converse: "If $x^2 = y^2$, then $x = y$."

Crucial Conceptual Insight for Exams

★ THE GOLDEN RULE: Just because a Proposition is TRUE, its Converse is NOT automatically True

• Truth Value Independence:

1. Proposition True, Converse True: e.g., "If $x = y$, then $x + a = y + a$" (Converse: "If $x + a = y + a$, then $x = y$"). Both are True.

2. Proposition True, Converse False: e.g., "If a shape is a square, then its angles are 90°" (Converse: "If all angles are 90°, it is a square" — False, could be a Rectangle!).

How to Disprove False Statements Instantly

• Definition: A counterexample is a single specific case or example where the Hypothesis (If part) holds true, but the Conclusion (Then part) fails.

• Power of Counterexample: In mathematics, to prove a statement true, you must prove it generally. But to prove a statement FALSE, finding just ONE counterexample is enough

• Practical Example:

- Statement: "If n is divisible by 2 and 4, then n is divisible by 8."

- Counterexample: Take n = 12.

*** 12 is divisible by 2 (True) and 4 (True).**

*** Is 12 divisible by 8? NO! (12 / 8 = 1.5).**

- Conclusion: The statement is completely FALSE.

Summary

1. PROPOSITION ——► "If X, then Y" (Statement: True or False)

2. CONVERSE ————► Swap X and Y ——► "If Y, then X"

3. GOLDEN RULE ——► Proposition = True DOES NOT MEAN Converse = True

4. COUNTEREXAMPLE ► ONE single valid bad case proves a statement FALSE

Practice questions

MULTIPLE CHOICE QUESTIONS (3 Questions)

For video solution youtube link : https://youtu.be/6uNtfUvr6xk(concepts)

https://youtu.be/QjHPErmzzJo (ch 9 imp questions)

Q1. Consider the proposition: "If a quadrilateral is a square, then all four of its angles are right angles." Which of the following is its converse, and what is its truth value?

(a) "If a quadrilateral is a square, then all its sides are equal." — True

(b) "If all four angles of a quadrilateral are right angles, then it is a square." — False

(c) "If all four angles of a quadrilateral are right angles, then it is a square." — True

(d) "If a quadrilateral is not a square, then its angles are not right angles." — False

Q2. If x and y are real numbers, consider the proposition P: "If x = y, then $x^2 = y^2$" and its converse Q. Which of the following statements is correct?

(a) Both P and Q are True.
(b) P is False, but Q is True.
(c) P is True, but Q is False.
(d) Both P and Q are False.

Q3. If a and b are perfect squares, then their product ab is a perfect square.

Which of the following examples disproves the converse of this statement?

(i) a = 4, b = 9, because ab = 36 is a perfect square.

(ii) a = 9, b = 25, because ab = 225 is a perfect square.

(iii) a = 4, b = 16, because ab = 64 is a perfect square.

(iv) a = 2, b = 8, because ab = 16 is a perfect square, but neither a nor b is a perfect square.

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DESCRIPTIVE & LOGICAL QUESTIONS (4 Questions)

Q4. Consider the divisibility statement: "If a positive integer n is divisible by 24, then it is divisible by both 4 and 6."

(i) Justify whether the statement is True or False.
(ii) Frame its converse and determine its truth value using a suitable counterexample.

Q5. Find counterexamples to disprove each of the following claims:

(i) Claim 1: "For every positive integer n, the number $(4n^2 + 1)$ is a prime number."
(ii) Claim 2: "If n is a prime number, then $(2^n - 1)$ is always a prime number."

Q6. Consider the geometric proposition: "If two triangles are congruent, then they have equal areas."

(i) Frame the converse of this proposition.
(ii) Is the converse true? Justify your answer with a concrete counterexample.

Q7. Consider the proposition: "If a quadrilateral is a rhombus, then its diagonals bisect each other at right angles (90°)."

(i) Frame the converse of this proposition.

(ii) Determine whether the converse is True or False, and justify your answer.

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HINTS & SOLUTIONS MASTER KEY

Q1. Solution & Steps [CBSE Concept Standard]

Hint: To form the converse of 'If X then Y', swap the condition and conclusion to get 'If Y then X'.

1. Proposition: "If a quadrilateral is a square (X), then all four of its angles are right angles (Y)."
2. Converse: "If all four angles of a quadrilateral are right angles (Y), then it is a square (X)."
3. Truth Value Check: A rectangle with unequal adjacent sides (e.g., 4 cm × 6 cm) has four right angles, but it is NOT a square.
4. Hence, the converse is FALSE.

Answer: (b) "If all four angles of a quadrilateral are right angles, then it is a square." — False

Q2. Solution & Steps [CBSE Board Trap MCQ]

Hint: Squaring preserves equality, but taking square roots introduces positive and negative possibilities.

1. Proposition P: If $x = y \Rightarrow x^2 = y^2$. (TRUE)
2. Converse Q: If $x^2 = y^2 \Rightarrow x = y$.
3. Counterexample for Q: Let $x = 3$ and $y = -3$.
   $x^2 = (3)^2 = 9$ and $y^2 = (-3)^2 = 9$. So $x^2 = y^2$ is True.
   However, $3 \neq -3$, so $x = y$ is FALSE.

Answer: (c) P is True, but Q is False.

Q3. Solution & Steps [CBSE Number Theory MCQ]

Hint: Test if two non-square numbers can multiply to produce a perfect square.

1. Converse Statement: "If ab is a perfect square, then both a and b are perfect squares."
2. Testing a = 2 (non-square) and b = 8 (non-square):
   Product ab = 2 × 8 = 16 = 4^2 (A PERFECT SQUARE).
3. Since ab is a perfect square but neither a nor b is a perfect square, the converse is FALSE.

Answer: (iv)

Q4. Solution & Steps [CBSE Board 3-Marker]

Hint: Joint divisibility by non-coprime numbers does not guarantee divisibility by their product.

1. (i) Proposition Justification: $24 = 4 \times 6$. Any multiple of 24 can be written as $24k = 4(6k) = 6(4k)$. Thus, it is divisible by both 4 and 6. Statement is TRUE.
2. (ii) Converse: "If a positive integer n is divisible by both 4 and 6, then it is divisible by 24."
3. Truth Value of Converse: FALSE.
   Counterexample: Take n = 12.
   • 12 is divisible by 4 (12 / 4 = 3)
   • 12 is divisible by 6 (12 / 6 = 2)
   • BUT 12 is NOT divisible by 24 (12 / 24 = 0.5).

Answer: (i) True, (ii) Converse is False (Counterexample: n = 12).

Q5. Solution & Steps [CBSE HOTS 3-Marker]

Hint: Test consecutive integer and prime values systematically until the expression yields a composite number.

1. (i) Claim 1: Testing values for $(4n^2 + 1)$:
   • n = 1 => $4(1)^2 + 1 = 5$ (Prime)
   • n = 2 => $4(2)^2 + 1 = 17$ (Prime)
   • n = 3 => $4(3)^2 + 1 = 37$ (Prime)
   • n = 4 => $4(4)^2 + 1 = 4(16) + 1 = 65$.
   Since $65 = 5 \times 13$ (Composite), n = 4 is the counterexample.

2. (ii) Claim 2: Testing prime values for $(2^n - 1)$:
   • n = 2 (Prime) => $2^2 - 1 = 3$ (Prime)
   • n = 3 (Prime) => $2^3 - 1 = 7$ (Prime)
   • n = 5 (Prime) => $2^5 - 1 = 31$ (Prime)
   • n = 11 (Prime) => $2^{11} - 1 = 2048 - 1 = 2047$.
   Since $2047 = 23 \times 89$ (Composite), n = 11 is the counterexample.

Answer: (i) Counterexample n = 4 (yields 65), (ii) Counterexample n = 11 (yields 2047).

Q6. Solution & Steps [CBSE Logical Geometry]

Hint: Congruence requires identical dimensions, whereas area depends only on (1/2) × base × height.

1. (i) Converse: "If two triangles have equal areas, then they are congruent."
2. (ii) Truth Value of Converse: FALSE.
3. Counterexample:
   • Consider $\triangle 1$ with Base = 10 cm and Height = 4 cm. Area = $(1/2) \times 10 \times 4 = 20$ cm².
   • Consider $\triangle 2$ with Base = 8 cm and Height = 5 cm. Area = $(1/2) \times 8 \times 5 = 20$ cm².

• Both triangles have equal areas (20 cm²), but their corresponding sides are not equal, so they are NOT congruent.
Answer: (i) "If two triangles have equal areas, then they are congruent.", (ii) False (Triangles with base-height pairs (10,4) and (8,5) have equal area 20 cm² but are non-congruent).

Q7. Solution & Steps [CBSE Geometry Property]

Hint: Recall the standard quadrilateral properties regarding diagonal bisectors at right angles.
1. (i) Converse Statement: "If the diagonals of a quadrilateral bisect each other at right angles (90°), then it is a rhombus."
2. (ii) Truth Value Check: TRUE.
• Justification: Let ABCD be a quadrilateral with diagonals AC and BD bisecting each other perpendicularly at point O (OA = OC, OB = OD, ∠AOB = 90°).
• In △AOB and △COB: OA = OC (given), ∠AOB = ∠COB = 90°, OB = OB (common).
• By SAS Congruence Rule, △AOB ≅ △COB, which implies AB = BC.
• Similarly, all four sides AB = BC = CD = DA are equal. Therefore, the quadrilateral is a rhombus.
Answer: (i) "If the diagonals of a quadrilateral bisect each other at right angles, then it is a rhombus.", (ii) True.

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