MATHS BASEMAKER

Class 10 CBSE — Chapter 12: Surface Areas and Volumes

Chapter Test Paper (Strictly Non-NCERT Board & Exemplar Selection)

SECTION A: MULTIPLE CHOICE QUESTIONS (4 Questions)

Q1. If the radius of the base of a right circular cone is halved and its height is doubled, then the ratio of the volume of the new cone to that of the original cone is:

(a) 1 : 2 (b) 2 : 1

(c) 1 : 4 (d) 4 : 1

Q2. The largest possible sphere is carved out of a solid wooden cube of side 21 cm. The volume of the remaining wood is (take π = 22/7):

(a) 4851cm³ (b) 4410 cm³

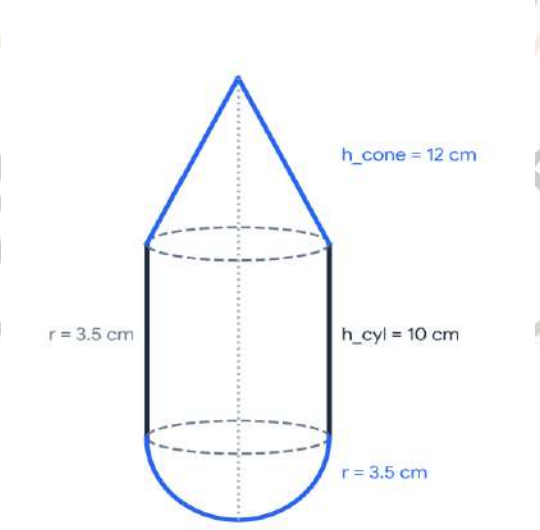
(c) 5240 cm³ (d) 3860 cm³

Q3. A hollow hemispherical metallic bowl has internal and external radii of 6 cm and 8 cm respectively. If the entire bowl (inside, outside, and the top circular rim) is painted, the total surface area to be painted is:

(a) 196π cm² (b) 200π cm²

(c) 256π cm² (d) 228π cm²

Q4. In the given figure, a solid toy is formed by a cylinder of base radius 3.5 cm and height 10 cm, surmounted by a cone of height 12 cm at the top and a hemisphere of the same radius at the bottom. The total height of the toy is:



(a) 25.5 cm (b) 22.0 cm

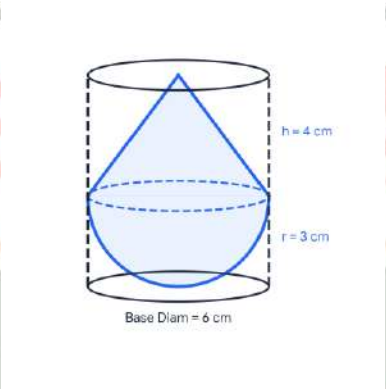
(c) 29.0 cm (d) 24.5 cm

DESCRIPTIVE QUESTIONS

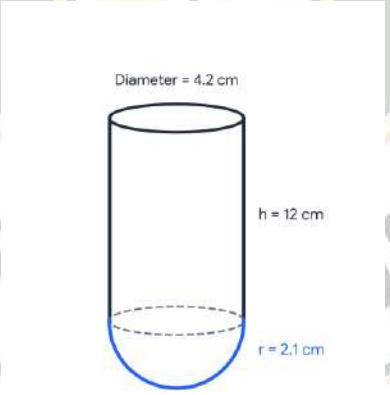
Q5. A circus tent is cylindrical up to a height of 3 m and conical above it. If the base diameter of the tent is 105 m and the slant height of the conical roof is 53 m, calculate the total area of the canvas required to make the tent. (take π = 22/7)

Q6. Solid metallic cones, each of base diameter 4 cm and height 3 cm, are melted and recast into a single solid sphere of diameter 12 cm. Find the total number of cones melted.

Q7. In the given figure, a solid toy is in the form of a hemisphere surmounted by a right circular cone of the same base diameter 6 cm. The height of the conical part is 4 cm..
If a right circular cylinder circumscribes the toy, find the volume of air in between toy and cylinder . (take π = 3.14)

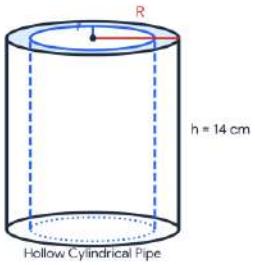


Q8. In the given figure, a glass test tube has a hemispherical bottom of diameter 4.2 cm and a cylindrical upper portion of height 12 cm.
(i) Find the liquid capacity (volume) of the test tube.
(ii) Find the internal surface area of the test tube. (take π = 22/7)



Q9. In the given figure, a metallic hollow cylindrical pipe is 14 cm long. The difference between its outside and inside curved surface areas is 44 cm^2. If the pipe is made of 99 cm^3 of metal, find the outer radius R and inner radius r of the pipe. (take π = 22/7)

Youtube link for video solutions: https://youtu.be/ZogI2WkNZ6I



Q10. Water is flowing at the rate of 2 m/s through a circular pipe of internal diameter 3.5 cm into a large cylindrical storage tank of diameter 1.4 m and height 2.1 m. Calculate the exact time (in minutes) required to completely fill the tank. (take π = 22/7)

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MATHS BASEMAKER
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HINTS & STEPS

Q1. Solution & Steps [CBSE Board Standard MCQ]

Hint: Volume of a cone = (1/3)πr²h. Substitute the new dimensions r' = r/2 and h' = 2h.

1. Original volume $V_1 = (1/3)\pi r^2 h$.
2. New radius r' = r/2, New height h' = 2h.
   New volume $V_2 = (1/3)\pi(r')^2 h' = (1/3)\pi(r/2)^2(2h) = (1/3)\pi(r^2/4)(2h) = (1/2) \times [(1/3)\pi r^2 h] = (1/2)V_1$.
3. Ratio of new volume to original volume = $V_2 : V_1 = (1/2) : 1 = 1 : 2$.

Answer: (a) 1 : 2

Q2. Solution & Steps [CBSE Board MCQ]

Hint: The diameter of the largest carved sphere equals the edge of the cube: 2r = 21 cm => r = 10.5 cm. Remaining volume = Volume of cube - Volume of sphere.

1. Side of cube a = 21 cm => Volume of cube = $a^3 = 21^3 = 9261$ cm³.
2. Diameter of sphere = 21 cm => Radius r = 21/2 cm.
   Volume of sphere = $(4/3)\pi r^3 = (4/3) \times (22/7) \times (21/2)^3 = 4851$ cm³.
3. Volume of remaining wood = 9261 - 4851 = 4410 cm³.

Answer: (b) 4410 cm³

Q3. Solution & Steps [CBSE Board Trap MCQ]

Hint: Total surface area of hollow hemisphere = Inner CSA + Outer CSA + Area of top ring = $2\pi r^2 + 2\pi R^2 + \pi(R^2 - r^2) = \pi(3R^2 + r^2)$.

1. Internal radius r = 6 cm, External radius R = 8 cm.
2. Inner CSA = $2\pi r^2 = 2\pi(6^2) = 72\pi$ cm².
3. Outer CSA = $2\pi R^2 = 2\pi(8^2) = 128\pi$ cm².
4. Top circular rim area = $\pi(R^2 - r^2) = \pi(8^2 - 6^2) = \pi(64 - 36) = 28\pi$ cm².
5. Total area to be painted = $72\pi + 128\pi + 28\pi = 228\pi$ cm².

Answer: (a) 228π cm²

Q4. Solution & Steps [CBSE Official Sample Paper MCQ]

Hint: Total height is the sum of the vertical heights of all three joined components along the central axis.

1. Height of conical top = 12 cm.
2. Height of cylindrical body = 10 cm.
3. Depth of hemispherical bottom = radius = 3.5 cm.
4. Total height of the toy = 12 + 10 + 3.5 = 25.5 cm.

Answer: (a) 25.5 cm

Q5. Solution & Steps [CBSE Board 3-Marker]

Hint: Canvas required = Curved surface area of cylinder + Curved surface area of cone = $2\pi rh + \pi rl = \pi r(2h + l)$.

1. Base diameter = 105 m => Radius r = 105/2 = 52.5 m.

Height of cylinder h = 3 m, Slant height of cone l = 53 m.

2. CSA of cylinder = $2\pi rh = 2 \times (22/7) \times (105/2) \times 3 = 22 \times 15 \times 3 = 990\ m^2$.

3. CSA of conical roof = $\pi rl = (22/7) \times (105/2) \times 53 = 11 \times 15 \times 53 = 8745\ m^2$.

4. Total canvas required = $990 + 8745 = 9735\ m^2$.

Answer: 9735 m^2

Q6. Solution & Steps [CBSE Board 3-Marker]

Hint: Number of cones = Volume of solid sphere / Volume of one cone.

1. Solid cone: diameter = 4 cm => r = 2 cm, height h = 3 cm.

Volume of 1 cone = $(1/3)\pi r^2h = (1/3) \times \pi \times 2^2 \times 3 = 4\pi\ cm^3$.

2. Recast sphere: diameter = 12 cm => radius R = 6 cm.

Volume of sphere = $(4/3)\pi R^3 = (4/3) \times \pi \times 6^3 = 288\pi\ cm^3$.

3. Number of cones n = Volume of sphere / Volume of 1 cone = $(288\pi) / (4\pi) = 72$ cones.

Answer: 72 cones

Q7. Solution & Steps [CBSE Board 4-Marker HOTS]

Hint: Volume of toy = Volume of cone + Volume of hemisphere. The circumscribing cylinder has base radius equal to the toy and height equal to total height of the toy.

1. Radius of cone and hemisphere r = 6/2 = 3 cm, Height of cone h = 4 cm.

2. Volume of conical part = $(1/3)\pi r^2h = (1/3) \times 3.14 \times 3^2 \times 4 = 37.68\ cm^3$ (or $12\pi\ cm^3$).

Volume of hemispherical part = $(2/3)\pi r^3 = (2/3) \times 3.14 \times 3^3 = 56.52\ cm^3$ (or $18\pi\ cm^3$).

Volume of toy = $37.68 + 56.52 = 94.2\ cm^3$ (or $30\pi\ cm^3$).

3. Circumscribing cylinder: base radius R = 3 cm, height H = h_cone + r_hemi = 4 + 3 = 7 cm.

Volume of cylinder = $\pi R^2H = 3.14 \times 3^2 \times 7 = 197.82\ cm^3$ (or $63\pi\ cm^3$).

4. Difference of volumes = $197.82 - 94.2 = 103.62\ cm^3$ (or $33\pi\ cm^3$).

Answer: 103.62 cm^3 (or $33\pi\ cm^3$)

Q8. Solution & Steps [CBSE Board Exam Classic 4-Marker]

Hint: Test tube capacity = Volume of cylinder + Volume of hemisphere. Internal surface area = CSA of cylinder + CSA of hemisphere.

1. Diameter = 4.2 cm => Radius r = 2.1 cm. Height of cylinder h = 12 cm.

2. Capacity (Volume) = $\pi r^2h + (2/3)\pi r^3 = \pi r^2(h + 2r/3)$

$= (22/7) \times (2.1)^2 \times [12 + 2(2.1)/3] = 13.86 \times 13.4 = 185.724\ cm^3$.

3. Internal surface area = $2\pi rh + 2\pi r^2 = 2\pi r(h + r)$

$= 2 \times (22/7) \times 2.1 \times (12 + 2.1) = 13.2 \times 14.1 = 186.12\ cm^2$.

Answer: (i) 185.724 cm^3, (ii) 186.12 cm^2

Q9. Solution & Steps [CBSE Board 4-Marker Logical — Hollow Pipe]

Hint: Difference of curved surface areas gives an equation in (R - r). Volume of metal gives an equation in ($R^2 - r^2$). Solve the resulting linear system.

1. Let the outer radius be R and inner radius be r. Height of pipe h = 14 cm.
2. Difference between outside and inside curved surface areas:

   $2\pi Rh - 2\pi rh = 44$

   $2 \times (22/7) \times 14 \times (R - r) = 44$

   $88 \times (R - r) = 44 \Rightarrow R - r = 44 / 88 = 0.5$ cm --- (1)
3. Volume of metal used in the hollow pipe:

   $\pi(R^2 - r^2)h = 99$

   $(22/7) \times (R - r)(R + r) \times 14 = 99$

   $44 \times (R - r)(R + r) = 99$

   Substitute (R - r) = 0.5 from equation (1):

   $44 \times 0.5 \times (R + r) = 99 \Rightarrow 22 \times (R + r) = 99$

   $R + r = 99 / 22 = 4.5$ cm --- (2)
4. Adding equations (1) and (2):

   $(R - r) + (R + r) = 0.5 + 4.5$

   $2R = 5.0 \Rightarrow R = 2.5$ cm.
5. Substituting R = 2.5 in equation (1):

   $2.5 - r = 0.5 \Rightarrow r = 2.5 - 0.5 = 2.0$ cm.

Answer: Outer radius R = 2.5 cm, Inner radius r = 2.0 cm

Q10. Solution & Steps [CBSE Board 5-Marker HOTS]

Hint: Volume discharged per second = Area of pipe cross-section × Speed. Time required = Volume of storage tank / Volume discharged per second.

1. Storage tank: diameter = 1.4 m => Radius R = 0.7 m = 70 cm, Height H = 2.1 m = 210 cm.

   Volume of tank = $\pi R^2 H = \pi \times 70 \times 70 \times 210$
2. Pipe: internal diameter = 3.5 cm => radius r = 1.75 cm.

   Speed of water = 2 m/s = 200 cm/s.

   Rate of water flow per second = $\pi r^2 \times$ speed = $\pi \times (1.75)^2 \times 200$.
3. Time taken to fill tank = $\pi \times 70 \times 70 \times 210 / \pi \times (1.75)^2 \times 200 = 1680$ seconds.
4. Converting to minutes: 1680 / 60 = 28 minutes exact.

Answer: 28 minutes (exact)

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