MATHS BASEMAKER

Class 9 CBSE — Ganita Manjari Part 2: Chapter 10

How Quantities Combine: Understanding Data (Comprehensive Guide & Master Test Paper)

PART 1: CHAPTER BASICS & CORE CONCEPTS

1. Simple Average vs. Weighted Average

- Simple Average: Used when all individual items or groups have equal size/importance.
  Formula: Simple Mean = (Sum of all observations) ÷ (Total number of observations)
- Weighted Average: Used when combining groups of different sizes or metrics with different weights/importance.
  Formula: Weighted Mean = $(w_1x_1 + w_2x_2 + ... + w_nx_n) \div (w_1 + w_2 + ... + w_n)$
- Common Mistake: Taking the simple average of two group averages (e.g., averaging 70 and 80 to get 75) is incorrect if the group sizes differ!

2. Mixture & Concentration Calculations

- Concentration = (Amount of solute / Pure substance) ÷ (Total volume of solution) × 100%
- When mixing multiple solutions of volumes V_1, V_2,... with concentrations C_1, C_2,...:
  Combined Concentration = $(V_1C_1 + V_2C_2 + ...) \div (V_1 + V_2 + ...)$
- Alligation Principle: The concentration of a mixture of two components always lies strictly between the concentrations of the individual components.

3. Ancient Indian Gold Purity Metric (Varṇa System)

- In ancient India (Pāṭīgaṇita by Śrīdharācārya, c. 750 CE), gold purity was measured in Varṇa (out of 16 base units).
- Relation to modern Karat: 16 Varṇa = 24 Karat (Pure Gold).
- When combining gold pieces of weights w_i and purities k_i (in Varṇa):
  Combined Purity (in Varṇa) = $(w_1k_1 + w_2k_2 + ... + w_nk_n) \div (w_1 + w_2 + ... + w_n)$

4. Updating Moving Averages & Reconstructing Totals

- Reconstructing Total Sum: Total Sum = (Average) × (Number of Observations, n)
- New Average after adding a new observation x_new: New Mean = (Old Total + x_new) ÷ (n + 1)
- New Average after removing observation x_old: New Mean = (Old Total - x_old) ÷ (n - 1)

5. Graphical Representations & Data Visualization

- Grouped Frequency Tables: Converting inclusive class intervals (e.g., 101–110) into continuous boundaries (100.5–110.5).
- Stacked Bar Chart vs. 100% Stacked Bar Chart:
  - Stacked Bar Chart shows actual total magnitudes split by sub-categories.
  - 100% Stacked Bar Chart standardizes all totals to 100% to compare proportional shares (relative frequencies).

https://youtu.be/PvJadjYTXeM (Video solutions of imp questions)

https://youtu.be/ljugoDNKs0g **(Video solutions of concepts)**

PART 2: TEN IMPORTANT QUESTIONS FOR EXAMS

Q1. The average score of students on a mathematics test in Section A is 72, and that of Section B is 76. Calculate the combined average score of both sections given that Section A has 30 students and Section B has 25 students.

Q2. A goldsmith melts together three pieces of gold alloy: 9 units at 12 varṇa purity, 5 units at 10 varṇa purity, and 17 units at 11 varṇa purity. Calculate the exact purity in varṇa of the combined gold alloy.

Q3. A farmer mixes three equal quantities of fertilisers. The first contains 1/10 fraction of nitrogen, the second contains 9/50 fraction of nitrogen, and the third contains 3/60 fraction of nitrogen. What is the fraction of nitrogen in the final mixture?

Q4 A restaurant collects customer ratings across three metrics: Food (average score 4.3), Ambience (average score 3.3), and Service (average score 2.8). If the management combines these metrics with weights Food : Ambience : Service = 6 : 5 : 4, calculate the overall weighted average rating of the restaurant.

Q5 .Batch A: Weighs 100 kg and contains 60% copper.

Batch B: Weighs 150 kg and contains 40% copper.

Batch C: Contains 30% copper.

When all three batches are melted together, the final combined alloy contains 40% copper. Find the weight of Batch C.

Q6. The average daily distance covered by a migratory bird over its first 20 days is 44.5 km.
(i) If it travels 55 km on the 21st day, calculate its new 21-day mean daily distance.
(ii) If on the 22nd day it rests (covers 0 km), find the updated 22-day mean daily distance.

Q7. In an animal research facility, the average weight of male langurs is 16.5 kg and that of female langurs is 13.8 kg. The overall combined average weight of all langurs is 14.925 kg. Given that the total count of langurs is 60 then find the number of male and female langurs in the facility.

Q8. In a 20-over cricket match, a team has a run rate of 6.0 runs per over after 19 overs. If they score 12 runs in the 20th over, calculate their final combined run rate.

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HINTS & STEP-BY-STEP SOLUTIONS

Q1.

Hint: Weighted Mean = $(n_1x_1 + n_2x_2) \div (n_1 + n_2)$. Do not take simple average (72+76)/2!

1. Total score of Section A = 72 × 30 = 2160.
2. Total score of Section B = 76 × 25 = 1900.
3. Combined Total Score = 2160 + 1900 = 4060.
4. Combined Total Students = 30 + 25 = 55.
5. Combined Average = 4060 ÷ 55 = 812 / 11 ≈ 73.82.

Answer: 73.82 (or 812/11)

Q2.

Hint: Combined Purity = $(\Sigma w_i k_i) \div (\Sigma w_i)$. Pure gold = 16 Varṇa.

1. Total units of alloy = 9 + 5 + 17 = 31 units.
2. Total varṇa-units = (9 × 12) + (5 × 10) + (17 × 11)
   = 108 + 50 + 187 = 345.
3. Combined Purity = 345 ÷ 31 ≈ 11.129 varṇa.

Answer: 11.13 Varṇa (approx)

Q3.

Hint: Since quantities are equal, take simple average of the three fractions.

1. Nitrogen fractions: F_1 = 1/10 = 10/100, F_2 = 9/50 = 18/100, F_3 = 3/60 = 1/20 = 5/100.
2. Sum of nitrogen fractions = 10/100 + 18/100 + 5/100 = 33/100.
3. Since 3 equal quantities are mixed, Fraction in mixture = (33/100) ÷ 3 = 11/100.

Answer: 11/100 (or 11%)

Q4.

Given weight ratio:

Food : Ambience : Service = 6 : 5 : 4

Total weight = 6 + 5 + 4 = 15

Therefore:

- Food importance = (6/15) × 100 = 40%
- Ambience importance = (5/15) × 100 = 33.33%
- Service importance = (4/15) × 100 = 26.67%

This means Food contributes 40%, Ambience 33.33% and Service 26.67% to the overall rating.

Weighted Average = Sum of (Rating × Weight) / Total Weight

= [(4.3 × 6) + (3.3 × 5) + (2.8 × 4)] / (6 + 5 + 4)

= (25.8 + 16.5 + 11.2) / 15

= 53.5 / 15

= 3.57 out of 5

Q 5 Let the weight of Batch C be **x** kg.

1. Calculate Pure Copper Content:

- From Batch A = 0.60 × 100 = **60 kg**
- From Batch B = 0.40 × 150 = **60 kg**
- From Batch C = 0.30 × x = **0.30x kg**
- Total Pure Copper = 60 + 60 + 0.30x = **120 + 0.30x**

2. Calculate Total Mixture Weight:

- Total Weight = 100 + 150 + x = **250 + x**

3. Formulate and Solve Equation: Since the final combined alloy contains 40% copper: Total Pure Copper = 0.40 × (Total Weight)

120 + 0.30x = 0.40 × (250 + x)

120 + 0.30x = 100 + 0.40x

120 - 100 = 0.40x - 0.30x

20 = 0.10x

x = 20 / 0.10

x = 200 kg

Answer: Weight of Batch C = **200 kg**

Q6.

Hint: New Total = Old Total + New Distance. New Mean = New Total ÷ New Number of Days.

1. Distance in first 20 days = 44.5 × 20 = 890 km.

2. (i) Distance on 21st day = 55 km.

   New 21-day Total = 890 + 55 = 945 km.

   New Mean = 945 ÷ 21 = 45 km/day.

3. (ii) Distance on 22nd day = 0 km.

   New 22-day Total = 945 + 0 = 945 km.

   New 22-day Mean = 945 ÷ 22 ≈ 42.95 km/day.

Answer: (i) 45 km/day, (ii) 42.95 km/day

Q7.

Hint: Let x = male langurs, y = female langurs. x + y = 60. Combined avg = (16.5x + 13.8y)/60 = 14.925.

1. (i) 16.5x + 13.8y = 14.925 × 60 = 895.5.

   Also 13.8x + 13.8y = 13.8 × 60 = 828.

   Subtracting gives 2.7x = 67.5 => x = 25 male langurs.

   Female langurs y = 60 - 25 = 35 female langurs.

Answer: 25 Males, 35 Females

Q8.

Hint: Run Rate = Total Runs / Total Overs. Stacked bar charts compare relative shares.

1. (i) Runs in first 19 overs = 19 × 6.0 = 114 runs.

   Scored in 20th over = 12 runs. Total runs in 20 overs = 114 + 12 = 126 runs.

   Final Run Rate = 126 ÷ 20 = 6.30 runs per over.

2. **Answer: 6.30 runs/over**

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